

Width of the $J^P = \frac{1}{2}^+$ pentaquark in the quark-diquark model

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We analyse the width of the $\theta(\frac{1}{2}^+)$ pentaquark assuming that it is a bound state of two extended spin-zero ud -diquarks and the \bar{s} antiquark (the Jaffe-Wilczek scenario). The width obtained when the size parameters of the pentaquark wave function are taken close to the parameters of the nucleon is found to be $\simeq 150$ MeV, i.e. it has a normal value for a P -wave hadron decay with the corresponding energy release. However, we found a strong dynamical suppression of the decay width if the pentaquark has an asymmetric "peanut" structure with the strange antiquark in the center and the two diquarks rotating around. In this case a decay width of $\simeq 1$ MeV is a natural possibility.

1. INTRODUCTION

A narrow hadron of the suspected quark content $udud\bar{s}$, the Pentaquark $\theta(1540)$, has been reported by several experiments [1]. In addition, a narrow charmed exotic state with the presumed content $udud\bar{c}$ has been observed [2]. Nevertheless, there still remain doubts because of the negative results from some other experiments [3] and problems with the identification of pentaquark states with different flavours [4, 5]. A striking feature of the pentaquark is its width of only several MeV [6]. The pentaquark has most likely spin $1/2$ and isospin 0. Its parity is not yet known.

Several interpretations of this state have been discussed in the literature (see [4, 7] and refs therein). Most likely it is the narrow exotic state predicted by Diakonov, Petrov and Polyakov [8] using a chiral soliton model. For a discussion of the width obtained in this model and of earlier suggestions [9] we refer to [10].

Our goal is to study the width of the new state in a quark model. To this end we follow the suggestion of Jaffe and Wilczek [11] that the correlation between two quarks in a 0^+ color antitriplet state is of particular importance. Their pentaquark model consists therefore of two scalar isospin zero diquarks in a relative angular momentum $L = 1$ state and the strange antiquark. The parity of this state is even. In our view this configuration is very likely since the correlation between two quarks in a color antitriplet state forming scalar diquarks is as strong as the corresponding $\bar{q}q$ correlation in a π meson. This is known from QCD sum rules [12] and from instanton calculations [13]. The most dramatic effect of diquark formation manifests itself in strangeness changing weak decays at low energies: It causes the huge $\Delta I = 1/2$ enhancement present in these processes [14].

The $\theta \rightarrow NK$ decay amplitude can be obtained from the single constant g_A , the $\theta \rightarrow N$ axial vector form factor at $q^2 = m_K^2$. In the analogous nucleon-to-nucleon transition one has $g_A^N(q^2 = 0) \simeq 1.25$. For a P -wave resonance with a mass of 1540 MeV the value $g_A = 0.8$ would lead to the width $\Gamma(\theta) = \Gamma(\theta \rightarrow pK^0) + \Gamma(\theta \rightarrow nK^+) = 160$ MeV. To have $\Gamma(\theta) \leq 10$ MeV one needs a strong suppression $g_A \leq 0.2$. Several recent papers have discussed possible reasons for a suppression of the $\theta \rightarrow NK$ transition amplitude by describing the spin-flavour and color factors in the $\langle NK|\theta \rangle$ overlap amplitude [7, 15, 16, 17, 18]. But a complete model calculation has so far not been performed.

We calculate the width of the pentaquark in a non-relativistic Fock space description using three and five constituent quarks for the proton and the pentaquark, respectively. A non-relativistic treatment seems appropriate since the spatial momentum of the final nucleon (in the rest system of the pentaquark) is not large. We use the quark-diquark picture for the proton and the pentaquark taking the size of the diquarks similar to the pion size. This appears to be justified from the near equality of the decay constants of pion and diquark and from the agreement of the proton decay constant using a spatially extended diquark with other determinations [12, 14]. The essential ingredients in our calculation are the size parameters of the pentaquark wave function for the diquark-diquark-antiquark configuration. If all size parameters of the $\frac{1}{2}^+$ pentaquark wave function are taken to be close to the size parameters of the nucleon, we obtain the width $\Gamma(\theta) \simeq 150$ MeV. On the other hand, the width is found to be very strongly suppressed when the pentaquark has a significantly asymmetric structure ("peanut"-like), in which the strange antiquark remains near to the center and the two diquarks are rotating around it. A width of $\simeq 1$ MeV is then easily obtainable.

We can also conclude from our investigation that any strong suppression of the transition amplitude must have a dynamical origin in the spatial part of the pentaquark wave function. We did not find a sizeable suppression of the decay amplitude due to the color-flavour structure of the pentaquark.

The paper is organized as follows: In the next section we discuss the relationship between the $\theta \rightarrow NK$ amplitude and the form factor g_A obtained from the axial-vector matrix element $\langle N|\bar{s}\gamma_\mu\gamma_5 d|\theta \rangle$. In section 3 we describe the nucleon and the pentaquark in terms of quark creation operators and calculate g_A in terms of the corresponding wave

functions according to the quark diquark picture. Section 4 contains the numerical study of g_A and the width for various choices of the parameters for the pentaquark wave function. Section 5 presents our conclusions.

2. THE $\theta \rightarrow KN$ DECAY AND THE FORM FACTOR g_A

The formula for the two-body decay width of the θ resonance has the form

$$\Gamma(\theta \rightarrow KN) = \frac{1}{8\pi} \frac{|\vec{q}|}{M_\theta^2} \sum_{\text{final spins}} |T(\theta \rightarrow KN)|^2, \quad (2.1)$$

where the sum runs over nucleon polarizations and $|\vec{q}|$ denotes the momentum of the outgoing nucleon in the θ rest frame

$$|\vec{q}| = \frac{1}{2M_\theta} \sqrt{(M_\theta^2 - M_N^2 - M_K^2)^2 - 4M_N^2 M_K^2}. \quad (2.2)$$

The decay amplitude $T(\theta \rightarrow KN)$ is related to the S -matrix according to

$$\langle N(p') K(q) | S | \theta(p) \rangle = i(2\pi)^4 \delta(p - p' - q) T(\theta \rightarrow NK) \quad (2.3)$$

for the standard relativistic normalization of states $\langle \vec{p} | \vec{p}' \rangle = 2p^0 (2\pi)^3 \delta(\vec{p} - \vec{p}')$. The amplitude may be expressed by the $\theta \rightarrow N$ matrix element of an interpolating field ϕ_K of the kaon

$$T(\theta \rightarrow NK) = \lim_{q^2 \rightarrow M_K^2} (M_K^2 - q^2) \langle N | \phi_K(0) | \theta \rangle. \quad (2.4)$$

Without loss of generality, ϕ_K can be taken to be the divergence of the strangeness-changing axial-vector current A_μ :

$$A_\mu = \bar{s} \gamma_\mu \gamma_5 d^a, \quad \phi_K = \frac{1}{f_K M_K^2} \partial^\mu A_\mu, \quad (2.5)$$

where f_K denotes the kaon decay constant ($f_K = 0.160$ GeV). The matrix element of the axial current between the θ and the nucleon can be decomposed in terms of invariant form factors

$$\langle N(p') | \bar{s} \gamma_\mu \gamma_5 d | \theta(p) \rangle = g_A(q^2) \bar{u}_N(p') \gamma_\mu \gamma_5 u_\theta(p) + g_P(q^2) q_\mu \bar{u}_N(p') \gamma_5 u_\theta(p) + g_T(q^2) \bar{u}_N(p') \sigma_{\mu\nu} q^\nu \gamma_5 u_\theta(p) \quad (2.6)$$

with $q = p - p'$. The spinor amplitudes for both θ and N are normalized according to relation $\bar{u}_i(p) u_i(p) = 2M_i$, $i = N, \theta$. The form factors g_i contain poles at $q^2 > 0$ due to strange meson resonances with the appropriate quantum numbers: e.g. g_P contains the K -meson pole and g_A the K_A^* pole.

In the chiral limit $m_s = m_d = M_K = 0$, the axial current is conserved $\partial_\mu A^\mu = 0$ and the form factors g_A and g_P are related to each other. The form factor $g_P(q^2)$ has a pole at $q^2 = 0$ corresponding to the massless kaon

$$g_P(q^2) = \frac{M_\theta + M_N}{q^2} g_A(q^2). \quad (2.7)$$

For $m_s \neq 0$ and $M_K \neq 0$, the pole in the form factor g_P is shifted to M_K^2 , and the form factors g_A and g_P are no longer related to each other by (2.7). However, due to the smallness of the strange-quark mass the main effect of its nonzero value results in the shift of the location of the pole in q^2 . We then have the approximate relation

$$g_P(q^2) = -\frac{M_\theta + M_N}{M_K^2 - q^2} g_A(q^2), \quad (2.8)$$

and in particular the PCAC result

$$\langle N(p') | \partial_\mu A^\mu | \theta(p) \rangle = \bar{u}_N(p') i \gamma_5 u_\theta(p) (M_\theta + M_N) g_A(q^2) \frac{M_K^2}{M_K^2 - q^2}. \quad (2.9)$$

Therefore, the decay amplitude is known if we know $g_A = g_A(M_K^2)$:

$$T(\theta \rightarrow NK) = \frac{M_\theta + M_N}{f_K} g_A \cdot \bar{u}_N(p') i \gamma_5 u_\theta(p). \quad (2.10)$$

The decay width (2.1) takes the simple form

$$\Gamma(\theta \rightarrow K^+ n) = \Gamma(\theta \rightarrow K^0 p) = \frac{1}{2\pi} \frac{|\vec{q}|^3}{f_K^2} \frac{(M_\theta + M_N)^2}{(M_\theta + M_N)^2 - M_K^2} g_A^2 \simeq \frac{1}{2\pi} \frac{|\vec{q}|^3}{f_K^2} g_A^2. \quad (2.11)$$

For $M_\theta = 1540$ MeV one finds $|\vec{q}| = 270$ MeV. The corresponding width is $\Gamma(\theta) = \Gamma(\theta \rightarrow K^+ n) + \Gamma(\theta \rightarrow K^0 p) \simeq 240 g_A^2$ MeV. Typically, for transitions between hadrons of the same quark structure one has $g_A \simeq 1$ (e.g. for the nucleon $g_A \simeq 1.23$), such that for a normal resonance with mass $M = 1540$ MeV one would expect $\Gamma(\theta(1540) \rightarrow KN) \simeq 120 \div 180$ MeV [10]. To obtain a width of ≤ 10 MeV one needs a strongly suppressed value $g_A \leq 0.2$.

We shall obtain the form factor g_A by calculating different components of the l.h.s. of (2.6) for different polarizations of the initial θ , working in the θ rest frame $p = (M_\theta, \vec{0})$ and choosing $q = (q_0, 0, 0, |\vec{q}|)$. It is convenient to use now the following normalization of states

$$\langle \vec{p} | \vec{p}' \rangle = (2\pi)^3 \delta(\vec{p} - \vec{p}'). \quad (2.12)$$

Then g_A can be obtained from the equation

$$g_A(q^2) = (M_N - M_\theta) \frac{(M_\theta + M_N)^2 - q^2}{4M_\theta^2} F_0 + \frac{M_N + M_\theta}{2M_\theta} A_L - \frac{q^2(A_T - A_L)}{(M_N - M_\theta)^2 - q^2} \quad (2.13)$$

with

$$F_0 = \frac{1}{|\vec{q}|} \langle N^\uparrow(\vec{p}') | \bar{s} \gamma_0 \gamma_5 d | \theta^\uparrow \rangle, \quad A_L = \langle N^\uparrow(\vec{p}') | \bar{s} \gamma_3 \gamma_5 d | \theta^\uparrow \rangle, \quad A_T = i \langle N^\uparrow(\vec{p}') | \bar{s} \gamma_2 \gamma_5 d | \theta^\perp \rangle. \quad (2.14)$$

$\vec{p}' = -\vec{q}$ lies in the negative z -direction, $|\vec{q}| = \sqrt{(M_\theta^2 - M_N^2 - q^2)^2 - 4M_N^2 q^2} / (2M_\theta)$. Note that $A_L = A_T$ for $\vec{q} = 0$.

In a model calculation of the left-hand side of (2.6) one cannot expect to obtain the K -meson pole. This means that not all the components of the current, and therefore not all form factors can be calculated in a quark model for the nucleon and the pentaquark. However one can expect that the linear combination of the current components relevant for the form factor g_A , which does not have a pole in the physical $\theta \rightarrow N$ decay region (the K_A^* resonance which leads to the pole in g_A lies at a much higher scale), can be calculated reliably within the quark model.

3. THE $\theta \rightarrow p K^0$ AMPLITUDE FROM A NONRELATIVISTIC QUARK MODEL

In this section we calculate the amplitudes $\langle N(-\vec{q}) | \bar{s} \gamma_\mu \gamma_5 d | \theta \rangle$ and the form factor $g_A(q^2)$ using a non-relativistic equal-time Fock space representation. To do so we replace the strange quark field s by its charge conjugate field $s_c = C(\bar{s})^T$ which annihilates the strange antiquark in θ . $C = i\gamma_2 \gamma_0$ is the charge conjugation matrix.

To make notations simpler we imply that the operator \hat{s} creates the \bar{s} -quark from the vacuum, and the operators u and d create u and d quarks, respectively.

Let us describe now the states involved in the decay, specifically for the process $\theta \rightarrow p K^0$.

A. The nucleon

In terms of three constituent quarks the proton state can be written in the form [12]

$$|N^\uparrow(\vec{p})\rangle = \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 e^{i\vec{p}\vec{R}} \Psi_N(\vec{r}_1 | \vec{r}_2, \vec{r}_3) u^{a\uparrow}(\vec{r}_1) D^a(\vec{r}_2, \vec{r}_3) |0\rangle, \quad (3.1)$$

where we have introduced a bilocal creation operator D for two quarks in a color-antitriplet state:

$$D^a(\vec{r}_2, \vec{r}_3) = \epsilon^{aa_2 a_3} [u^{a_2\uparrow}(\vec{r}_2) d^{a_3\downarrow}(\vec{r}_3) - u^{a_2\downarrow}(\vec{r}_2) d^{a_3\uparrow}(\vec{r}_3)] \quad (3.2)$$

and

$$\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3). \quad (3.3)$$

The radial wave function $\Psi_N(r_1 | r_2, r_3)$ must be symmetric under $2 \leftrightarrow 3$ to guarantee that the nucleon has spin and isospin 1/2. It depends on the relative coordinates ρ and λ

$$\vec{\rho} = \vec{r}_2 - \vec{r}_3, \quad \vec{\lambda} = \frac{1}{2}(\vec{r}_2 + \vec{r}_3) - \vec{r}_1. \quad (3.4)$$

i.e.

$$\Psi_N(r_1|r_2, r_3) = \Psi_N(\vec{\rho}, \vec{\lambda}). \quad (3.5)$$

The state normalization (2.12) leads to the following condition for the proton radial wave function [12]

$$6 \int d\vec{\rho} d\vec{\lambda} \Psi_N(\vec{\rho}, \vec{\lambda}) \left[2\Psi_N(\vec{\rho}, \vec{\lambda}) + \Psi_N\left(\frac{1}{2}\vec{\rho} - \vec{\lambda}, -\frac{3}{4}\vec{\rho} - \frac{1}{2}\vec{\lambda}\right) \right] = 1. \quad (3.6)$$

We parametrize the wave function by harmonic oscillator functions in ρ^2 and λ^2 which correspond to assuming harmonic forces between the constituents:

$$\Psi_N(\rho, \lambda) = \frac{1}{\sqrt{N_p}} \exp\left(-\frac{1}{2\alpha_{\rho N}^2}\rho^2 - \frac{2}{3\alpha_{\lambda N}^2}\lambda^2\right). \quad (3.7)$$

The normalization factor N_p is determined by (3.6). A fully symmetric function under coordinate exchanges corresponds to $\alpha_{\rho N} = \alpha_{\lambda N}$.¹ If $\alpha_{\rho N} \neq \alpha_{\lambda N}$, a mixed-symmetry component is generated in the nucleon, but existing quark potential models do not support such a contribution of a probability larger than $1 \div 2\%$ [19]. As can be seen from Eqs. (3.1) and (3.3), $\alpha_{\rho N}$ is the size parameter of the diquark for which we will use the notation α_D .

The wave function (3.7) leads to the following expression for the proton electromagnetic form factor

$$F_p(Q^2) = \frac{2}{3} \exp\left(-\frac{Q^2\alpha_{\lambda N}^2}{12}\right) + \frac{1}{3} \exp\left(-\frac{Q^2}{48}(\alpha_{\lambda N}^2 + 3\alpha_{\rho N}^2)\right), \quad Q^2 = \vec{q}^2. \quad (3.8)$$

The derivative of the form factor at $Q^2 = 0$ is experimentally known to be close to $1/M_\rho^2$, with M_ρ denoting the ρ -meson mass. Thus we get a constraint on the parameters $\alpha_{\rho N}$ and $\alpha_{\lambda N}$

$$-F'_p(0) = \frac{\alpha_{\lambda N}^2}{16} + \frac{\alpha_{\rho N}^2}{48} \simeq \frac{1}{M_\rho^2}. \quad (3.9)$$

For the fully symmetric wave function one gets $\alpha_{\rho N} = \alpha_{\lambda N} = 2\sqrt{3}/M_\rho = 0.9$ fm. The nonrelativistic formula (3.8) gives a good description of the proton form factor in the region $Q^2 = 0 \div 0.3$ GeV², relevant for our analysis.

B. The pentaquark

As described in the introduction, we consider the pentaquark as consisting of two spin and isospin zero diquarks in a relative $L = 1$ angular momentum, and a strange antiquark \bar{s} . Thus we take the following form for the $J = 1/2$ pentaquark state

$$\begin{aligned} |\theta^\uparrow(\vec{p})\rangle &= \int d\vec{r}_1 d\vec{r}_2 d\vec{r}_3 d\vec{r}_4 d\vec{r}_5 \exp\left(i\vec{p} \frac{m_s \vec{r}_1 + m \vec{r}_2 + m \vec{r}_3 + m \vec{r}_4 + m \vec{r}_5}{m_s + 4m}\right) \\ &\times \epsilon^{abc} \left[\sqrt{\frac{2}{3}} \hat{s}^{a\downarrow}(\vec{r}_1) \Psi_{1,1} - \frac{1}{\sqrt{3}} \hat{s}^{a\uparrow}(\vec{r}_1) \Psi_{1,0} \right] D^b(\vec{r}_2, \vec{r}_3) D^c(\vec{r}_4, \vec{r}_5) |0\rangle. \end{aligned} \quad (3.10)$$

Here we introduced again the creation operator $D^a(\vec{r}_i, \vec{r}_j)$ of two correlated quarks. The radial wave functions $\Psi_{1,i}$ are angular momentum 1 eigenstates with projection i to the z -axis. The function $\Psi_{1,i}$ should be symmetric under $2 \leftrightarrow 3$ and under $4 \leftrightarrow 5$ to provide zero values for the diquark spin and isospin, and antisymmetric under $\{23\} \leftrightarrow \{45\}$ (i.e. in diquark space coordinates). Let us now introduce the variables

$$\begin{aligned} \vec{r}_{23} &= \vec{r}_2 - \vec{r}_3, & \vec{R}_{23} &= \frac{1}{2}(\vec{r}_2 + \vec{r}_3), & \vec{r}_{45} &= \vec{r}_4 - \vec{r}_5, & \vec{R}_{45} &= \frac{1}{2}(\vec{r}_4 + \vec{r}_5), \\ \vec{\rho}_\theta &= \vec{R}_{23} - \vec{R}_{45}, & \vec{\lambda}_\theta &= \frac{1}{2}(\vec{R}_{23} + \vec{R}_{45}) - \vec{r}_1, \end{aligned} \quad (3.11)$$

¹ We emphasize that for the symmetric case the nucleon wave function (3.7) is fully equivalent to the standard SU(6) symmetric 56-plet wave function. By reordering it describes along with the spinless ud diquark made of quarks 2 and 3, the spin-1 ud "diquark" made of particles 1 and 3. The quark-diquark form of the nucleon wave function becomes more than just a notation if the correlation between the u and d quarks in the scalar state is stronger than that in the relative spin-1 state.

where \vec{r}_1 is the position of the strange particle, \vec{R}_{23} and \vec{R}_{45} are the positions of the two diquarks. Taking into account the above symmetry requirements, the P -wave radial wave function of the pentaquark can be written as follows

$$\begin{aligned}\Psi_{1,1}(r_1, r_2, r_3, r_4, r_5) &= i \frac{\rho_x + i y}{\sqrt{2}} \Psi_\theta(r_1 | r_2, r_3 | r_4, r_5), \\ \Psi_{1,0}(r_1, r_2, r_3, r_4, r_5) &= -i \rho_z \Psi_\theta(r_1 | r_2, r_3 | r_4, r_5),\end{aligned}\quad (3.12)$$

where Ψ_θ is the radial-symmetric part of the pentaquark wave function. The diquark picture then amounts to the following form for the latter

$$\Psi_\theta(r_1 | r_2, r_3 | r_4, r_5) = \Phi_\theta(\rho_\theta, \lambda_\theta) \Phi_D(r_{23}) \Phi_D(r_{45}), \quad (3.13)$$

Eq. (2.12) requires the normalization

$$\int |\Phi_D(r)|^2 d\vec{r} = 1, \quad 64 \int \vec{\rho}_\theta^2 |\Phi_\theta(\rho_\theta, \lambda_\theta)|^2 d\vec{\rho}_\theta d\vec{\lambda}_\theta = 1, \quad (3.14)$$

Again, we take a Gaussian parameterizations for the wave function

$$\Psi_\theta(r_1 | r_2, r_3 | r_4, r_5) = \frac{1}{\sqrt{N_\theta}} \exp\left(-\frac{1}{2\alpha_\rho^2} \rho_\theta^2 - \frac{2}{3\alpha_\lambda^2} \lambda_\theta^2\right) \exp\left(-\frac{\vec{r}_{23}^2}{2\alpha_D^2}\right) \exp\left(-\frac{\vec{r}_{45}^2}{2\alpha_D^2}\right), \quad (3.15)$$

and determine the normalization factor N_θ from (3.14).

C. The transition amplitude

In the non-relativistic approximation the quark current operators take the following form (summation over color indices is implied)

$$\begin{aligned}\bar{s}(\vec{x}_s) \gamma_0 \gamma_5 d(\vec{x}_d) &\rightarrow -\hat{s}^\dagger(\vec{x}_s) d^\dagger(\vec{x}_d) + \dots, \\ \bar{s}(\vec{x}_s) \gamma_3 \gamma_5 d(\vec{x}_d) &\rightarrow \left(\frac{k_x - i k_y}{2m_d} - \frac{k'_x - i k'_y}{2m_s}\right) \hat{s}^\dagger(\vec{x}_s) d^\dagger(\vec{x}_d) - \left(\frac{k'_z}{2m_s} + \frac{k_z}{2m_d}\right) \hat{s}^\dagger(\vec{x}_s) d^\dagger(\vec{x}_d) + \dots, \\ i\bar{s}(\vec{x}_s) \gamma_2 \gamma_5 d(\vec{x}_d) &\rightarrow \left(\frac{k_x - i k_y}{2m_d} - \frac{k'_x + i k'_y}{2m_s}\right) \hat{s}^\dagger(\vec{x}_s) d^\dagger(\vec{x}_d) + \left(\frac{k'_z}{2m_s} - \frac{k_z}{2m_d}\right) \hat{s}^\dagger(\vec{x}_s) d^\dagger(\vec{x}_d) + \dots\end{aligned}\quad (3.16)$$

Here \vec{k}' is the momentum operator of the s -quark, $\vec{k}' = -i\vec{\nabla}^{(s)}$; \vec{k} is the momentum operator of the d -quark, $\vec{k} = -i\vec{\nabla}^{(d)}$. The dots stand for structures which contain the operators $s^\dagger d^\dagger$ or $s^\dagger d^\dagger$. Their contributions to the $\theta \rightarrow N^\dagger$ transition amplitudes vanish: $\langle N^\dagger | s^\dagger d^\dagger | \theta \rangle = 0$ and $\langle N^\dagger | s^\dagger d^\dagger | \theta \rangle = 0$.

The $\theta \rightarrow N^\dagger$ transition amplitudes of the quark currents above may be expressed through

$$\vec{\mathcal{A}} = \frac{24}{\sqrt{3}} \int d\vec{r}_2 d\vec{r}_4 d\vec{r}_5 \exp\left(i\vec{q} \frac{\vec{r}_2 + \vec{r}_4 + \vec{r}_5}{3}\right) \vec{\rho}_\theta \Psi_\theta(r_s | r_2, r_d | r_4, r_5) \{2\Psi_N(r_2 | r_4, r_5) + \Psi_N(r_4 | r_2, r_5)\}, \quad (3.17)$$

with $\vec{\rho}_\theta = \frac{1}{2}(\vec{r}_2 + \vec{r}_d - \vec{r}_4 - \vec{r}_5)$ as in (3.11). The functions Ψ_N and Ψ_θ are given by Eqs. (3.5) and (3.13). The quantity $\vec{\mathcal{A}}$ follows after the sum over color and flavour is performed. The two terms in the curly brackets have the following origin: the first (with the factor 2) arises from the processes in which one of the diquarks is a spectator. The second term is due to the formation of the diquark in the proton by two quarks from different diquarks of the pentaquark.

The final result reads:

$$\begin{aligned}|\vec{q}|F_0 &= \langle N^\dagger | \bar{s} \gamma_0 \gamma_5 d | \theta^\dagger \rangle = -i \mathcal{A}_z |_{\vec{r}_s \rightarrow 0, \vec{r}_d \rightarrow 0} \\ A_L &= \langle N^\dagger | \bar{s} \gamma_3 \gamma_5 d | \theta^\dagger \rangle = \frac{1}{2m_s} \left\{ \frac{\partial}{\partial x_s} \mathcal{A}_x + \frac{\partial}{\partial y_s} \mathcal{A}_y + \frac{\partial}{\partial z_s} \mathcal{A}_z \right\} \\ &\quad + \frac{1}{2m_d} \left\{ -\frac{\partial}{\partial x_d} \mathcal{A}_x - \frac{\partial}{\partial y_d} \mathcal{A}_y + \frac{\partial}{\partial z_d} \mathcal{A}_z \right\} |_{\vec{r}_s \rightarrow 0, \vec{r}_d \rightarrow 0} \\ A_T &= i \langle N^\dagger | \bar{s} \gamma_2 \gamma_5 d | \theta^\dagger \rangle = \frac{1}{2m_s} \left\{ \frac{\partial}{\partial x_s} \mathcal{A}_x + \frac{\partial}{\partial y_s} \mathcal{A}_y + \frac{\partial}{\partial z_s} \mathcal{A}_z \right\} \\ &\quad + \frac{1}{2m_d} \left\{ -\frac{\partial}{\partial x_d} \mathcal{A}_x + \frac{\partial}{\partial y_d} \mathcal{A}_y - \frac{\partial}{\partial z_d} \mathcal{A}_z \right\} |_{\vec{r}_s \rightarrow 0, \vec{r}_d \rightarrow 0}.\end{aligned}\quad (3.18)$$

4. NUMERICAL ESTIMATES

We can give now our numerical results for the pentaquark width using (2.13) and (3.18). Two assumptions reduce the number of parameters:

1. The structure of the diquark in the nucleon and in the pentaquark coincide, i.e. we take the size-parameter α_D of the diquark wave function Φ_D to be equal to the parameter $\alpha_{\rho N}$ of the nucleon wave function

$$\alpha_D = \alpha_{\rho N}. \quad (4.1)$$

2. The parameters of the nucleon wave function are chosen such that the experimental nucleon electromagnetic form factor is reproduced for small momentum transfers,

$$\frac{\alpha_{\lambda N}^2}{16} + \frac{\alpha_D^2}{48} = \frac{1}{M_\rho^2}. \quad (4.2)$$

We first take a symmetric wave function $\alpha_{\lambda N} = \alpha_D$. As explained above, the diquark size parameter is then given by the relation $\alpha_D = \alpha_{\rho N} = 0.9$ fm, a number also compatible with the size of the pion. Now only the two free parameters of the pentaquark wave function remain to be fixed, namely, $\alpha_{\theta\rho}$ and $\alpha_{\theta\lambda}$. We study the dependence of g_A and of the width on these parameters. We recall that the average distance between the two diquarks is determined by $\alpha_{\theta\rho}$, and the average distance between the s -antiquark and the center-of-mass of the two diquarks is determined by $\alpha_{\theta\lambda}$.

The decay width involves the constituent quark masses m_s and $m_u = m_d$. We use the values $m_s \simeq 500$ MeV and $m_d \simeq 350$ MeV relevant for a nonrelativistic description. We have checked that the values $m_s \simeq 350$ MeV and $m_d \simeq 220$ obtained in a relativistic quark model [20] essentially lead to the same results, since the d and s quark contributions to the amplitudes partially compensate each other.

Little is known about the details of the pentaquark structure. We allow therefore the parameters $\alpha_{\theta\lambda}$ and $\alpha_{\theta\rho}$ to vary in a broad range

$$0.6 \text{ fm} < \alpha_{\theta\lambda}, \alpha_{\theta\rho} < 1.6 \text{ fm}. \quad (4.3)$$

If both parameters would be 1 fm, then $g_A \simeq 0.8$ and the corresponding width is 150 MeV. No suppression due to a possible mismatch of color and flavour quantum numbers in the initial and final states takes place.

However, a strong dynamical suppression occurs if the structure of the pentaquark is asymmetric. Figure 1 exhibits the dependence of g_A on the pentaquark size parameters. For instance, for

$$\alpha_{\theta\lambda} = 0.6 \text{ fm}, \quad \alpha_{\rho\lambda} = 1.4 \text{ fm}, \quad (4.4)$$

we get $g_A = 0.05$ and $\Gamma(\theta) = 1$ MeV. We note that a somewhat small $\alpha_{\theta\lambda}$ and a large $\alpha_{\theta\rho}$ imply an interesting "peanut" structure of the pentaquark: the two extended diquark balls overlap only partially, while the antiquark remains close to the center.

Figure 2 presents the form factor $g_A(q^2)$ for these values of the parameters. We observe that the form factor is a rather flat function of q^2 as expected from the location of the K_A^* resonance.

It is also of interest to see the influence of an extreme qq correlation (i.e. of a very small diquark size) on the pentaquark decay width. Figure 3 illustrates the pentaquark width vs the diquark size α_D for fixed values of the pentaquark size-parameters. It is seen that a sizeable reduction of the pentaquark width occurs only for a very small diquark size, which corresponds to an implausible large deviation from an SU(6)-symmetric nucleon wave function. The successful description of the nucleon using a symmetric nucleon wave function does not support such small-size diquarks. The diquark should be an extended object which dissolves into smaller partons in higher momentum transfer processes.

5. CONCLUSIONS

We studied the $\theta(\frac{1}{2}^+)$ pentaquark decay into NK final states, assuming that the pentaquark consists of two extended color-triplet ud diquarks with spin and isospin zero in a relative P -wave state, and a strange antiquark. The pentaquark and the nucleon were described in terms of non-relativistic Fock states with Gaussian spatial wave functions.

The pentaquark decay width $\Gamma(\theta)$ is found to depend strongly on the pentaquark configuration: when all size-parameters of the pentaquark wave function are close to 1 fm, one obtains a width of about 100 – 150 MeV, i.e. a typical hadronic value. The color-flavour structure of the pentaquark causes no suppression.

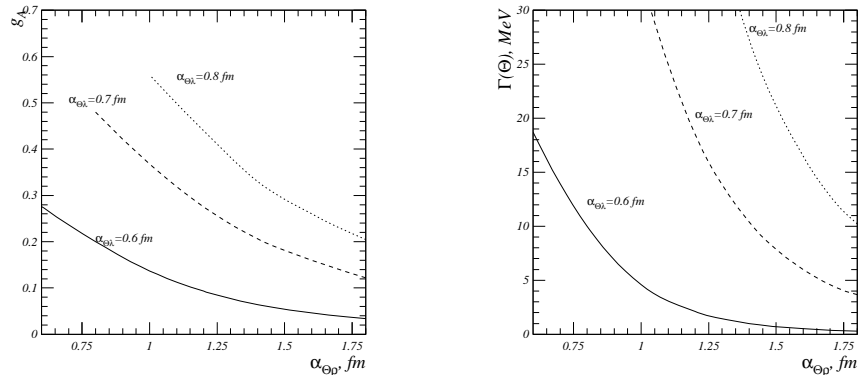


Fig. 1: The axial-vector coupling g_A and the decay width $\Gamma(\theta)$ vs the pentaquark size parameter $\alpha_{\theta\rho}$ for different values of $\alpha_{\theta\lambda}$.

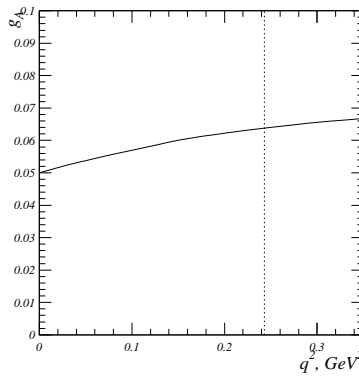


Fig. 2: The axial-vector form factor g_A vs q^2 for the pentaquark size parameters $\alpha_{\theta\rho} = 1.4$ fm and $\alpha_{\theta\lambda} = 0.6$ fm. The dotted line corresponds to $q^2 = M_K^2$.

However, a strong dynamical suppression of the amplitude occurs for a "peanut"-shaped pentaquark, i.e. when it has an asymmetric structure with $\alpha_{\theta\lambda} \ll \alpha_{\theta\rho}$. For instance, $\alpha_{\theta\lambda} = 0.6$ fm and $\alpha_{\theta\rho} = 1.4$ fm brings the width down to 1 MeV.

Because of the specific details of our analysis (non-relativistic treatment, Gaussian wave functions) the accuracy of our calculation is presumably only about 50%. Nevertheless, the proposed mechanisms for a strong suppression of the amplitude by a specific peanut-like structure of the pentaquark remains valid.

We conclude therefore that if the pentaquark can be described as a quark-diquark system, the small width requires a rather asymmetric structure with two extended diquarks rotating about the strange antiquark near the center.

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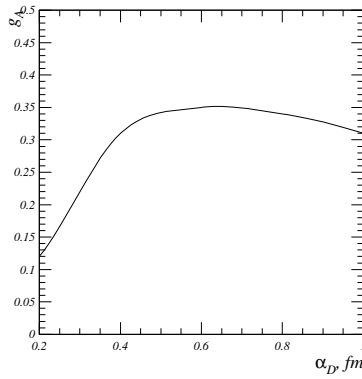


Fig. 3: The axial-vector coupling g_A vs the diquark size parameter $\alpha_D = \alpha_{N\rho}$. The parameter $\alpha_{N\lambda}$ of the nucleon wave function is calculated from $\alpha_{N\rho}$ using Eq. (3.9). The values of the pentaquark size parameters are fixed to be $\alpha_{\theta\rho} = 1.2$ fm and $\alpha_{\theta\lambda} = 1.0$ fm.

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